EFFECT OF SNOW POLLUTION

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Following from: *Analysis of solar heating of a snowpack*

A combination of physical processes of snow formation in the atmosphere and periodic partial melting of snow cover can lead to various morphologies of contaminated snow. Therefore, both the internal mixing of soot in ice grains and the external mixing when soot is between the grains should be considered. Following to (Dombrovsky and Kokhanovsky 2020), the geometrical optics (GO) approximation for ice grains, the Rayleigh theory for small soot particles, the Maxwell-Garnett theory for the effective optical constants of ice containing soot, and the Mie theory generalized for the case of two-layered spherical particles are employed in this article.

Due to very weak absorption of visible light by pure ice, even very small impurities may affect the absorption of light in the snow cover composed of ice grains. This effect is especially important in the Arctic regions located close to pollution sources (Doherty et al. 2010). The effect of various impurities on the spectral albedo of snow has been studied since the early paper by Warren and Wiscombe (1980), as well as in many relatively recent studies (Flanner et al. 2007, 2012, Aoki et al. 2011, Kokhanovsky 2013, Liou et al. 2014, Qian et al. 2015, Dang et al. 2015, He et al. 2018, Warren 2019). We restrict ourselves to considering the most important effect of soot, also called "black carbon," but the results obtained are qualitatively correct for other impurities.

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Two variants of location of soot particles are considered in the literature as well as in recent studies by Dombrovsky and Kokhanovsky (2019, 2020): inside the ice grains ("internally mixed") and between the grains ("externally mixed"). The case of externally mixed soot is simpler for direct modeling. However, some researchers consider the internally mixed soot as a very realistic case. The physical processes which lead to soot inclusion in ice particles were discussed by Flanner et al. (2012). It was estimated that 32–73% of soot particles in the snow are internally mixed with ice grains. Usually the uniform/random distribution of soot particles inside the ice grains is considered. However, the contributions of soot particles positioned in the central part or near the surface of an ice grain to the absorption of incident light are different (Dombrovsky 2000, 2002, 2004, Dombrovsky and Baillis 2010). The main objective of this article is to analyze the effect of a non-uniform distribution of soot in ice grains on the absorption coefficient of the polluted snow. The schematic of various spatial distributions of soot is presented in Fig. 1.

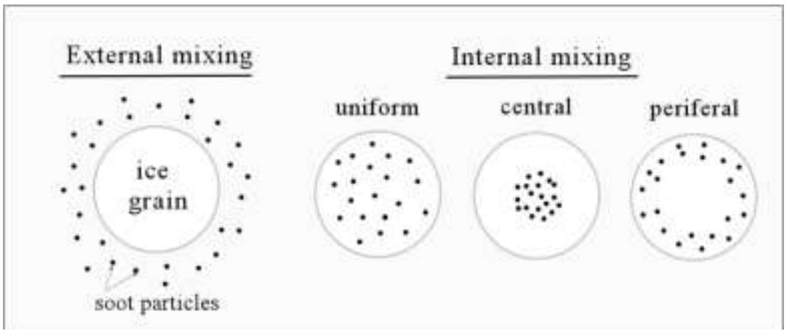

Figure 1. The schematic of various mixing of ice grains and soot particles.

In the case of external mixing, the absorption coefficient is additive: $\alpha_\lambda = \alpha_\lambda^{\text{pure}} + \alpha_\lambda^{\text{soot}}$. According to (Dombrovsky and Kokhanovsky 2020), we focus on light absorption by snow in the visible range and in the short-wavelength part of the near infrared. It is known that the index of absorption of ice in this range is very small and the analytical solution for spherical ice grains based on the GO approximation can be employed to calculate the optical properties of pure snow. It was shown by Dombrovsky et al. (2019) that this approach can be used instead of the rigorous Mie theory to calculate both the efficiency factor of absorption and transport albedo of single scattering for spherical ice grains in the spectral range of interest. As usual, our model is based on the hypothesis of independent scattering of light by single particles. The error of this approach is expected to be negligible because both the size of randomly positioned ice grains and the distances between them are much greater than the radiation wavelength. It should be also noted that the absorption coefficient of semi-transparent snow does not depend on the size of ice grains.

Soot particles are produced usually as a result of incomplete combustion of hydrocarbon fuels. Soot consists of nearly monodisperse spherical primary particles that collect into fractal aggregates having broad size distribution. The diameters of primary soot particles are usually in the range between 5 and 80 nm. Soot contains not only carbon but also hydrocarbons and other substances. As a result, the optical constants of soot may be different and do not coincide with optical constants of pure amorphous carbon. However, the effect of this difference on absorption coefficient of soot is insignificant (Dombrovsky and Baillis 2010).

The airborne soot is considered as the main component of the snow pollution. As a rule, atmospheric soot particles are characterized by a strong absorption in the visible range, but the radiation scattering by these particles is relatively small (Doner and Liu 2017, Wang et al. 2019). The calculation of radiation absorption by aggregates of primary soot particles is not a simple task.

The great uncertainty in the morphology of the aggregates makes reasonable the use of a simple model for physical estimates. The Rayleigh approximation for spherical soot particles is employed and the absorption coefficient of soot is expressed as follows (Dombrovsky and Baillis 2010):

$$\alpha_\lambda^{\text{soot}} = B\gamma_\lambda^{\text{soot}} f_v^{\text{soot}} \qquad B = \frac{9n_{\text{soot}}}{\left(n_{\text{soot}}^2 - \kappa_{\text{soot}}^2 + 2\right)^2 + 4n_{\text{soot}}^2\kappa_{\text{soot}}^2} \tag{1}$$

where $\gamma_\lambda^{\text{soot}} = 4\pi\kappa_{\text{soot}}/\lambda$ is the bulk absorption coefficient of soot, $f_{\text{v}}^{\text{soot}} = \rho_{\text{soot}}^{\text{cloud}}/\rho_{\text{soot}}$ is the volume fraction of soot ($\rho_{\text{soot}}^{\text{cloud}}$ is the density of the soot cloud, ρ_{soot} is the density of the soot particle substance), n_{soot} and κ_{soot} are the spectral indices of refraction and absorption of soot. The dispersion relation suggested by Dalzell and Sarofim (1969) for the optical constants of soot is used in subsequent calculations. According to Doherty et al. (2010, 2013), the local value of the mass fraction of soot far from populated areas is less than $s = 0.1$ppm (or 100 ng/g). However, the value of $s = 0.5$ ppm is also used below to estimate the role of relatively high pollution.

The effective medium approximation based on the Maxwell-Garnett theory is employed to determine the optical constants of ice containing small soot particles. According to this approach, the complex permittivity of a composite medium in ice grains is calculated in terms of particle polarizability by applying the Lorentz–Lorenz formula (Koledintseva et al. 2009, Markel 2016). The following relations are used to calculate the effective complex index of refraction $m_{\text{eff}} = n_{\text{eff}} - i\kappa_{\text{eff}}$ at the known values of $m = n - i\kappa$ for pure ice and $m_{\text{soot}} = n_{\text{soot}} - i\kappa_{\text{soot}}$ for soot:

$$m_{\text{eff}}^2 = m^2 \frac{2\delta_{\text{soot}}\left(m_{\text{soot}}^2 - m^2\right) + m_{\text{soot}}^2 + 2m^2}{2m^2 + m_{\text{soot}}^2 - \delta_{\text{soot}}\left(m_{\text{soot}}^2 - m^2\right)} \tag{2}$$

where δ_{soot} is the local volume fraction of soot in the ice grain. The volume fraction of soot is usually very small and one can rewrite Eq. (2) as follows:

$$m_{\text{eff}}^2 = m^2\left(1 + 3\delta_{\text{soot}}\frac{1 - \bar{m}^2}{1 + 2\bar{m}^2}\right) \qquad \bar{m} = m/m_{\text{soot}} \tag{3}$$

Equation (3) is preferable in the case of a combination of two very small values: δ_{soot} and κ. Note that a similar approach based on effective medium approximations was used by Flanner et al. (2012) to determine the optical constants of ice contaminated by soot particles. In all variants of the spatial distribution of soot particles in snow, the mass concentration of soot is assumed to be the same. It means that the local volume fraction of soot may be different. In the case of a spherical ice grain with uniform distribution of soot, the volume fraction of soot is determined as:

$$\delta_{\text{soot}}^0 = \frac{s}{f_{\text{v}}}\frac{\rho_{\text{ice}}}{\rho_{\text{soot}}} \tag{4}$$

where ρ_{ice} and ρ_{soot} are the densities of ice and soot, $f_v = \rho_{\text{snow}}/\rho_{\text{ice}}$ is the volume fraction of ice in snow (ρ_{snow} is the density of snow). The values of $\rho_{\text{ice}} = 916.7$kg/m^3, $\rho_{\text{soot}} = 2050$kg/m^3, and $f_{\text{v}} = 0.33$ are used in the calculations. The physical sense of Eq. (4) is quite clear because the ratio of s/f_{v} is the mass fraction of soot in the ice grain. It is interesting to consider two cases of nonuniform distribution of soot in a spherical ice grain (see Fig. 1): (1) The same mass of soot is uniformly distributed in the central part, $0 < r < a_c$ ($a_c \leq a$), of the ice grain with radius a; (2) All the soot is in the surface layer of the grain: $a - \Delta < r \leq a$. In these cases, the local value of δ_{soot} increases as follows:

$$\delta_{\mathrm{soot}} = \delta^0_{\mathrm{soot}}/\psi \qquad \psi = \begin{cases} \bar{a}_c^3, \text{ central pollution} \\ 1 - (1 - \bar{\Delta})^3, \text{peripheral pollution} \end{cases} \tag{5}$$

where $\bar{a}_c = a_c/a$ and $\bar{\Delta} = \Delta/a$. The values of $\bar{a}_c = 1$ and $\bar{\Delta} = 1$ correspond to the case of the uniformly distributed soot, when $\psi = 1$ and $\delta_{\mathrm{soot}} = \delta^0_{\mathrm{soot}}$. In other cases, $\psi < 1$ and $\delta_{\mathrm{soot}} > \delta^0_{\mathrm{soot}}$. Obviously, the soot particles increase the effective index of absorption of the polluted part of ice grain, whereas the effective index of refraction is insensitive to the presence of soot.

The computational estimates show that scattering properties of snow containing a small amount of soot are almost the same as those of pure snow, whereas the effect of soot on absorption coefficient of snow in the visible spectral range is significant. The efficiency factor of absorption, Q_a, of single particles is directly proportional to the radius a of homogeneous weakly absorbing particles. This makes convenient to work with the ratio of Q_a/a. In the case of snow composed of monodisperse spherical grains the spectral absorption coefficient of snow is determined as:

$$\alpha_\lambda = 0.75 f_v\, Q_a/a \tag{6}$$

where Q_a can be calculated not only for homogeneous spherical particles, but also for centrally symmetric inhomogeneous particles (Bohren and Huffman 1983, Dombrovsky 1996, Babenko et al. 2003). To solve the problem of the present article, it is sufficient to consider two-layered spherical particles. The corresponding solution and computer code can be found in the book by Bohren and Huffman (1983). Some modifications of this code are also freely available. The computer code by Dombrovsky (1996) was used in subsequent calculations. The results obtained for ice grains containing soot uniformly distributed in the central part or in the concentric surface layer of the grain at the wavelength of $\lambda = 0.5\mu$m and $s = 0.5$ppm are presented in Fig. 2. One can see some wave effects at $a = 20\mu$m and a fast transfer to the GO limit at $a \approx 50\mu$m with the universal (independent of a) monotonic dependences for the ice grains with $a \geq 50\mu$m. The latter is important because the radius of ice grains in a snowpack usually satisfies this condition. As a result, the absorption coefficient of snow containing polydisperse ice grains can be obtained using the above results for the monodisperse model. It is interesting that α_λ does not depend on $\bar{a}_c$ in the range of $\bar{a}_c < 0.75$ and decreases almost linearly at larger values of $\bar{a}_c$ (Fig. 2a). The effect of a predominant absorption in the central part of symmetrically illuminated large spherical particles have been studied in detail. The literature on this subject can be found in the book by Dombrovsky and Baillis (2010). The kick in the absorption profiles at radius of $r = a/n$ was explained by the geometrical optics effects. Moreover, the analysis of evolution of angular distribution of radiation intensity in the particle enabled to suggest a modified differential approximation used in some problems of nonuniform heating or cooling of semi-transparent particles (Dombrovsky 2002, Dombrovsky and Dinh 2008). The monotonic increase in the absorption coefficient with the relative thickness of the polluted surface layer of ice grains is also significant (Fig. 2b). In the case under consideration, we have $\alpha_\lambda = 1.6$ m^{-1} when soot is in a thin surface layer of the ice grain, $\alpha_\lambda = 3.3$ m^{-1} for the uniform distribution of soot in the grain, and $\alpha_\lambda = 4.4$ m^{-1} when soot is uniformly distributed in the central region of ice grain with radius $a_\mathrm{c} < 0.75a$. The difference between the above values should not be ignored in an analysis of absorption of visible radiation in the polluted snow cover.

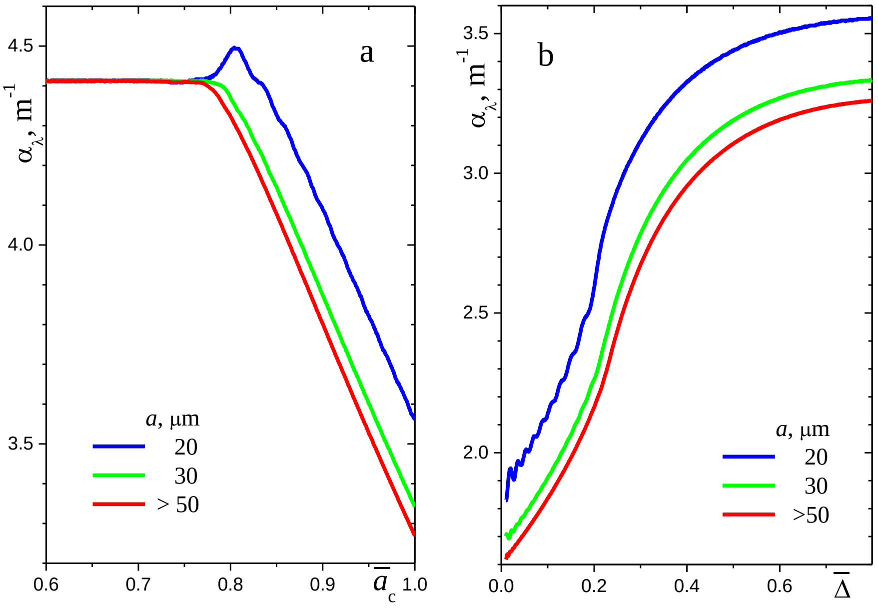

Figure 2. Absorption coefficient of snow with ice grains containing soot particles (a) in the central part or (b) in the surface layer of the grain.

The above calculations are based on the assumption of a symmetric irradiation of ice grains. Strictly speaking, this assumption is incorrect for the optically thin surface layer of snow. The asymmetric illumination of ice grains can be taken into account using the method developed by Dombrovsky (2004). However, this more accurate consideration is not necessary because of multiple scattering of visible solar radiation in snow and the resulting transformation of collimated solar radiation to the diffuse radiation in a thin surface layer of the snow cover.

Let us consider the spectral variation of the absorption coefficient of snow at various assumptions on distribution of soot in the ice grain. The results of calculations in the visible range and in the adjacent part of the near-infrared are presented in Fig. 3. The values of α_λ were obtained for $f_\mathrm{v} = 0.33$, but one can easily determine the absorption coefficient at another value of f_v. It is interesting that Mie theory calculations at $\bar{\Delta} = 0.01$ give almost the same absorption coefficient as that obtained for the external mixing of soot. This particular case should be taken into account while using the general statement by Liou et al. (2011) that “light-absorbing particles can absorb more radiation when they are mixed or coated with weakly-absorbing material”. Note that the latter statement is correct in the simplest variant of uniform distribution of soot in the ice grain.

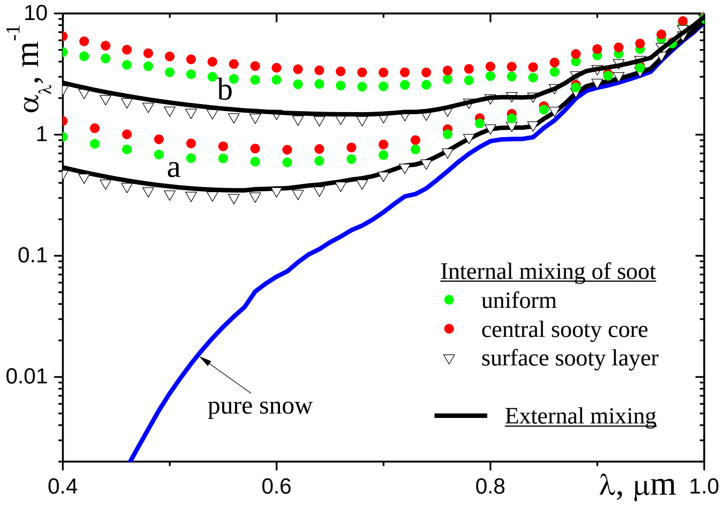

Figure 3. Effect of soot on absorption coefficient of snow: a – $s = 0.1$ppm, b – $s = 0.5$ppm.

The effect of vertically inhomogeneous polluted snow on solar heating and possible melting of a snowpack has been studied by Dombrovsky and Kokhanovsky (2019). It seems obvious that an additional solar heating due to absorption of solar radiation in a thin surface layer of not very polluted snow is partially compensated by the convective cooling. However, the additional solar heating may be significant when the layer of a dirty snow is thick or positioned at a distance under the surface because of partial thermal protection of deep layers from the convective cooling due low thermal conductivity of snow. It is interesting that some observations in Greenland reported by Doherty et al. (2010, 2013) showed the sharp maximum in concentration of BC positioned at distance of 10 cm under the snowpack surface.

Let us analyze the radiative transfer in a non-uniformly polluted snowpack. As compared with the general problem considered in the article *Analysis of solar heating of a snowpack*, we use two simplifications: the diffuse radiation from the sky is neglected and the snowpack is assumed to be horizontal. This makes it possible to focus on the effect of a non-uniform impurity of snow. Of course, the analytical solution derived for the uniform snowpack cannot be used. Instead, the known numerical procedure should be employed. To solve the problem, Dombrovsky and Kokhanovsky (2019) suggested a combined method based on dividing the whole spectrum in two parts. The numerical solution for a thick layer of snow containing one or several layers of snow with impurities is necessary only in the wavelength range of $\lambda < \lambda_* \approx 1\mu\text{m}$ because of the expected penetration of solar radiation into the deep layers. On the contrary, the radiation at $\lambda > \lambda_*$ is absorbed in a thin surface layer even in the case of pure snow. The latter enables us to use the analytical solution for the uniform snowpack of pure snow by ignoring the impurities. The heat transfer problem is exactly the same as that in the article *Analysis of solar heating of a snowpack*. It should be noted only that thermal conductivity of snow is taken equal to $k = 0.2$ W/(m K) and the maximum convective heat transfer coefficient in the middle of the day $h_{\max} = 10$ W/(m^2 K), which corresponds to a wind speed less that about 1–2 m/s. The calculated temperature profiles at different time moments during the first day are plotted in Fig. 4. The external mixing of soot at $s = 1$ppm is considered. The value of $\lambda_* = 1\mu$m was used in the calculations. One can see that the two-band spectral model is sufficiently accurate and can be used to estimate the thermal effects of layered impurities.

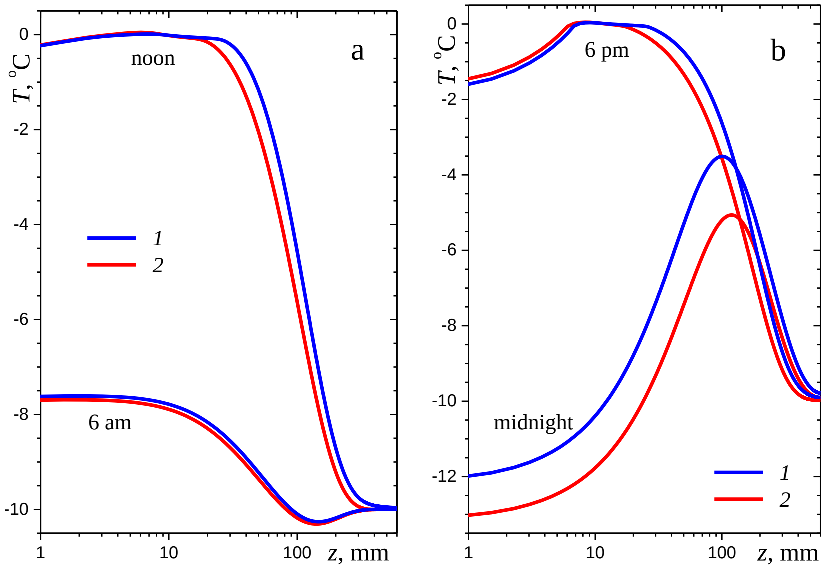

Figure 4. Typical profiles of temperature in snowpack (a) before the noon and (b) after the noon: *1* – analytical solution, *2* – two-band computational model.

The effect of thin but highly polluted ($s = 1$ppm) layers of snow on profiles of absorbed radiation power is illustrated in Fig. 5. The universal coordinates $f_v z$ and P/f_v are used to present simultaneously the results for various values of f_v. The increase in local absorption coefficient of snow leads to considerable variation of absorbed radiation power in the polluted layers and also in the deeper part of the snowpack. The effect of a single layer of soot-containing snow at $s = 1$ppm on temperature profiles at $f_v = 0.33$ is shown in Fig. 6. To minimize the effect of initial conditions, the second day from the beginning of heating is considered. The calculations show the melting of snow in the internal region of a snowpack at noon and also at 6 pm. The temperature profiles with a deeply positioned maximum at $z \approx 200$mm are observed at 6 pm. The temperature maximum at the midnight due to intense surface cooling is stronger in the case of a polluted snow. Note that a similar effect of the periodic formation of relatively warm and sufficiently thick layers in the ice cover leads to strong tensile stresses on the cold surface of ice. As a result, cracks and even large crevices can be formed on the surface of glaciers. Note that the latent heat of ice melting results in a considerable delay in snow melting and the so-called mushy zone at the melting temperature is formed during the melting. It is also important that heat is accumulated in a snowpack with time and the internal temperature increases day by day.

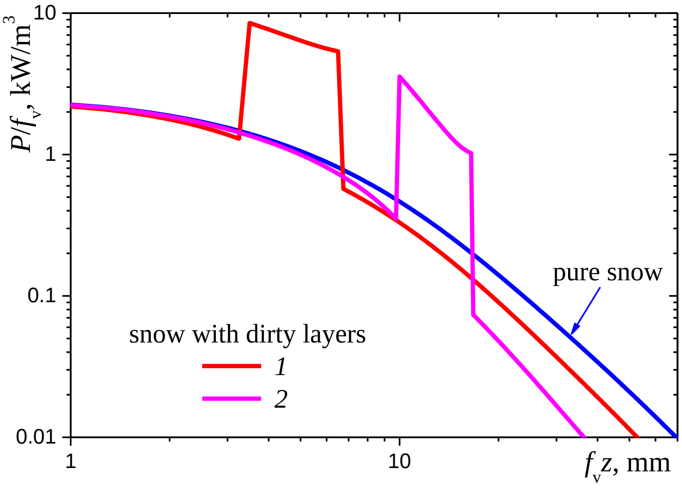

Figure 5. Effect of single layers of polluted snow on absorbed radiation power in the wavelength range of $\lambda < \lambda_*$: *1* – $3.3 < f_v z < 6.6$mm, *2* – $9.9 < f_v z < 13.2$mm.

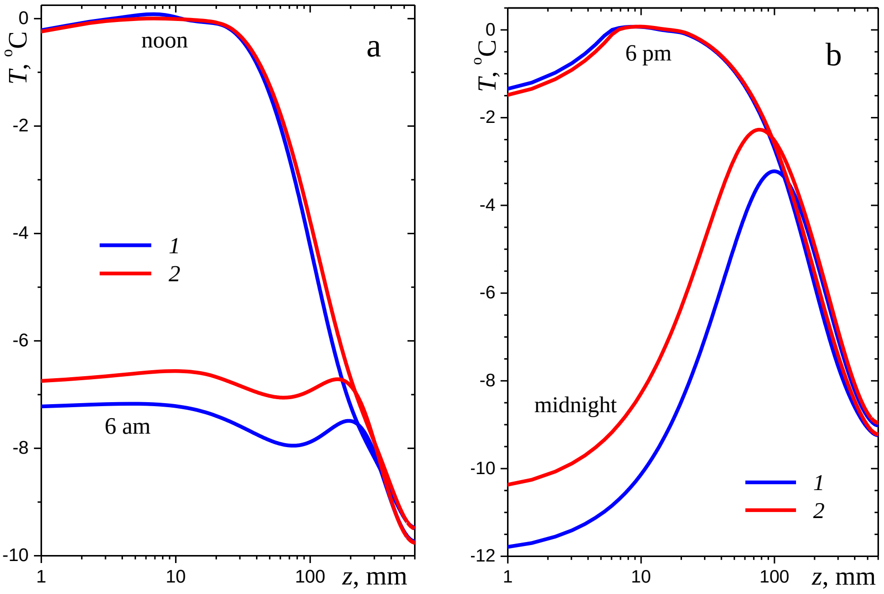

Figure 6. Effect of internal dirty layers on temperature profiles in a snowpack before the noon and (b) after the noon: *1* – pure snow; *2* – snow with polluted layer of $10 < z < 20$mm.

The results obtained showed that both the distribution of soot in ice grains and layered pollution of a snowpack affect the absorption of solar radiation and snowpack albedo in the visible range. The local layered impurities of snow may lead to considerable additional heating of deep layers of snow and the resulting accumulation of heat inside the snowpack.

References

Aoki, T., Kuchiki, K., Kodama, Y., Hosaka, M., and Tanaka, T. (2011) Physically Based Snow Albedo Model for Calculating Broadband Albedos and the Solar Heating Profile in Snowpack for General Circulation Models, *J. Geophys. Res. Atmos.*, 116 :D11114.

Babenko, V.A., Astafieva, L.G., and Kuzmin, V.N. (2003) *Electromagnetic Scattering in Disperse Media. Inhomogeneous and Anisotropic Particles*, Praxis, New York.

Bohren, C.F. and Huffman, D.R. (1983) *Absorption and Scattering of Light by Small Particles*, Wiley, New York.

Dalzell, W.H. and Sarofim, A.P. (1969) Optical Constants of Soot and Their Application to Heat-Flux Calculations, *ASME J. Heat Transf.*, 91 (1): 100-104.

Dang, C., Brandt, R.E., and Warren, S.J. (2015) Parametrization for Narrowband and Broadband Albedo of Pure Snow and Snow Containing Mineral Dust and Black Carbon, J. Geophys. Res. Atmos., 120 (11): 5446-5468.

Doherty, S.J., Warren, S.G., Grenfell, T.C., Clarke, A.D., and Brandt, R.E. (2010) Light-Absorbing Impurities in Arctic Snow, Atmos. Chem. Phys. 10 (23): 11647-11680.

Doherty, S.J., Warren, S.G., Forsström, S., Hegg, D.L., Brandt, R.E., and Warren, S.G. (2013) Observed Vertical Redistribution of Black Carbon and Other Insoluble Light-Absorbing Particles in Melting Snow, J. Geophys. Res. Atmos. 118 (11): 1-17.

Dombrovsky, L.A. (1996) *Radiation Heat Transfer in Disperse Systems*, Begell House, New York.

Dombrovsky, L.A. (2000) Thermal Radiation from Nonisothermal Spherical Particles of a Semitransparent Material, *Int. J. Heat Mass Transf.*, 43 (9): 1661-1672.

Dombrovsky, L.A. (2002) A Modified Differential Approximation for Thermal Radiation of Semitransparent Nonisothermal Particles: Application to Optical Diagnostics of Plasma Spraying. *J. Quant. Spectrosc. Radiat. Transf.*, 73 (2-5): 433-441.

Dombrovsky, L.A. (2004) Absorption of Thermal Radiation in Large Semi-Transparent Particles at Arbitrary Illumination of the Polydisperse System, *Int. J. Heat Mass Transf.*, 47 (25): 5511-5522.

Dombrovsky, L.A. and Baillis, D. (2010) *Thermal Radiation in Disperse Systems: An Engineering Approach*, Begell House, New York.

Dombrovsky, L.A. and Dinh, T.N. (2008) The Effect of Thermal Radiation on the Solidification Dynamics of Metal Oxide Melt Droplets, *Nucl. Eng. Design*, 238 (6): 1421-1429.

Dombrovsky, L.A. and Ignatiev, M.B. (2003) An Estimate of the Temperature of Semitransparent Oxide Particles in Thermal Spraying, *Heat Transf. Eng.*, 24 (2): 60-68.

Dombrovsky, L.A. and Kokhanovsky, A.A. (2019) The Influence of Pollution on Solar Heating and Melting of a Snowpack, *J. Quant. Spectrosc. Radiat. Transf.*, 233: 42-51.

Dombrovsky, L.A. and Kokhanovsky, A.A. (2020), Light Absorption by Polluted Snow Cover: Internal Versus External Mixture of Soot, *J. Quant. Spectrosc. Radiat. Transf.*, 242C: 106799.

Dombrovsky, L.A., Kokhanovsky, A.A., and Randrianalisoa, J.H. (2019) On Snowpack Heating by Solar Radiation: A Computational Model, *J. Quant. Spectrosc. Radiat. Transf.*, 227: 72–85.

Doner, N. and Liu, F. (2017) Impact of Morphology on the Radiative Properties of Fractal Soot Aggregates, *J. Quant. Spectrosc. Radiat. Transf.*, 187: 10-19.

Flanner, M.G., Zender, C.S., Randerson, J.T., and Rasch, P.J. (2007) Present-Day Climate Forcing and Response from Black Carbon in Snow, *J. Geophys. Res. Atmos.*, 112 (11): D11202.

Flanner, M.G., Liu, X., Zhou, C., Penner, J.E., and Jiao, C. (2012) Enhanced Solar Energy Absorption by Internally-Mixed Black Carbon in Snow Grains, *Atmos. Chem. Phys.*, 12 (10): 4699-4721.

He, C., Liou, K.-N., Takano, Y., Yang, P., Qi, L., and Chen, F. (2018) Impact of Grain Shape and Multiple Black Carbon Internal Mixing on Snow Albedo: Parametrization and Radiative Effect Analysis, *J. Geophys. Res. Atmos.*, 123 (2): 1253-1268.

Kokhanovsky, A. (2013) Spectral Reflectance of Solar Light from Dirty Snow: A Simple Theoretical Model and its Validation, *The Cryosphere*, 7 (4): 1325-1331.

Koledintseva, M.Y., DuBroff, R.E., and Schwartz, R.W. (2009) Maxwell Garnett Rule for Dielectric Mixtures with Statistically Distributed Orientations of Inclusions, *Prog. Electromag. Res.*, 99: 131-148.

Libois, Q., Picard, G., France, J.L., Arnaud, L., Dumont, M., Carmagnola, C.M., and King, M.D. (2013) Influence of Grain Shape on Light Penetration in Snow, *The Cryosphere*, 7 (6): 1803-1818.

Liou, K.N., Takano, Y., He, C., Yang, P., Leung, L.R., Gu, Y., and Lee, W.L. (2014) Stochastic Parametrization for Light Absorption by Internally Mixed BC/Dust in Snow Grains for Application to Climate Models, *J. Geophys. Res. Atmos.*, 119 (2): 7616-7632.

Lucarini, V., Saarinene, J.J., Peiponen, K.-E., and Vartainen, E.M. (2005) *Kramers–Krönig Relations in Optical Material Research, Springer Series in Optical Sciences*, Springer, Berlin.

Markel, V.A. (2016) Introduction to the Maxwell Garnett Approximation: Tutorial, *J. Opt. Soc. Am. A.*, 33 (7): 1244-1256.

Qian, Y., Yasunari, T.J., Doherty, S.J., Flanner, M.G., Lau, W.K.M., Jing, M., Wang, H., Wang, M., Warren, S.G., and Zhang, R. (2015) Light-Absorbing Particles in Snow and Ice: Measurement and Modeling of Climatic and Hydrological Impact, *Adv. Atmos. Sci.*, 32 (1): 64-91.

Wang, Y.-F., Huang, Q.-X., Wang, F., Chi, Y., and Yan, J.-H. (2019) A Feasible and Accurate Method for Calculating the Radiative Properties of Soot Particle Ensembles in Flames, *J. Quant. Spectrosc. Radiat. Transf.*, 224: 222-232.

Warren, S.G. (2019) Light-Absorbing Impurities in Snow: A Personal and Historical Account, *Front. Earth Sci.*, 6: 250.

Warren, S.G. and Clarke, A.D. (1990) Soot in the Atmosphere and Snow Surface of Antarctica, *J. Geophys. Res. Atmos.*, 95 (D2): 1811-1816.

Warren, S.G. and Wiscombe, W.J. (1980) A Model for the Spectral Albedo of Snow, II: Snow Containing Atmospheric Aerosols, *J. Atmos. Sci.*, 137 (12): 2734-2745.