SOLAR HEATING OF ICE CONTAINING BUBBLES

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Following from: *Computational models for combined heat transfer in snow and ice*

Numerous and diverse gas bubbles, often found in the volume of ice, continue to attract the attention of researchers for many years. These bubbles are clearly visible due to intense scattering of visible light. At the same time, the multiple scattering of light by gas bubbles significantly affects the absorption of solar light in semi-transparent ice layers. The study of the resulting increase in solar heating of ice sheets containing gas bubbles is one of the important tasks of geophysics. The radiative transfer in light-scattering ice have been studied during more than four past decades (Gilpin et al. 1977, Seki et al. 1979, Perovich and Grenfell 1982, Grenfell 1991, Mobley et al. 1998, McGuinness et al. 2001, Timofeev 2018, Sleptsov and Savvinova 2019, 2020, Dombrovsky and Kokhanovsky 2020). The ice heating and melting due to absorbed external radiation was also considered in some of these papers.

The presence of gas bubbles in various semi-transparent media affects the optical and thermal properties of such two-phase systems and hence the transport phenomena. One can mention the studies on influence scattering of light by bubbles in the near-surface layer of the ocean (Stamnes et al. 2017), the effect of bubbles generated by chemical reactions during the glass melting in industrial furnaces (Shelby 2005, Hrma 2009), and the optical properties of solid glass containing bubbles (Pilon and Viskanta 2003, Dombrovsky et al. 2005). Aerogels or other materials with numerous bubbles or hollow microspheres in semi-transparent host media are considered as advanced thermal insulations (Papadopoulos 2005, Dombrovsky 2005, Dombrovsky et al. 2007, Thapliyal and Singh 2014). In some cases, the pores in low-porous ceramics can be modeled as spherical gas bubbles (Manara et al. 1999, Lisitsyn et al. 2016). It was shown by Dombrovsky (2004) that the role of steam bubbles is considerable in radiative heating of boiling water. In this regard, it should be recalled that the problem of the fuel-coolant interaction in a hypothetical severe accident of nuclear reactors involves the near-infrared heating of water containing numerous bubbles (Dombrovsky 2007, 2009, Dombrovsky et al. 2009, Dombrovsky and Davydov 2010).

The methodological basis of recent study by Dombrovsky and Kokhanovsky (2020) on ice containing gas bubbles includes three physical problems: (1) determination of the optical properties of scattering ice, (2) spectral calculation of solar radiation transfer in a layer of the “cloudy ice”, and (3) the solution to heat transfer problem to determine the transient temperature profile in the ice sheet. These problems can be solved using the known analytical or numerical methods, so that the construction of a general computational model does not require special methodological developments. At the same time, the choice of particular approaches is limited by the main objective of the work, which is an analysis of the influence of the size and volume fraction of gas bubbles on heating of ice at a distance under the surface, when the volumetric solar heating becomes insensitive to convective heat transfer at the ice sheet surface.

The computational study is significantly simplified by using an analytical solution for the absorbed radiation power, since the numerical solution for the radiative transfer problem in an optically thick layer is associated with mathematical difficulties. Therefore, the physical problem statement contains a number of assumptions concerning both the optical properties of cloudy ice and the radiative transfer in ice sheet. The most important simplification is associated with multiple scattering of light in the spectral range of semi-transparency. In addition, the large optical thickness of the medium leads to a relatively simple angular dependence of the radiation intensity, which allows a transition to one of the differential approximations, as was done in (Gilpin et al. 1977, Perovich and Grenfell 1982, Sleptsov and Savvinova 2019, Dombrovsky and Kokhanovsky 2020). However, one should take into account the effect of total internal reflection on the ice/air interface.

Strictly speaking, even for the conditions of the polar summer, the time variation of solar zenith angle should not be ignored. Such detailed calculations were carried out by Dombrovsky et al. (2019) in computational analysis of solar heating of a snowpack. In the case of ice sheet containing gas bubbles, we restrict ourselves to calculations for the Sun at its zenith. The error of this assumption is not so great due to the refraction of sunlight on the surface of ice layer. Moreover, instead of a complex spectrum of solar radiation transmitted through the atmosphere, for a qualitative study of this work, the smooth Planck spectrum is considered.

The 1-D transient energy equation for temperature, $T(t,z)$, in the ice layer of thickness d and the accompanying initial and boundary conditions can be written as follows:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + (1-\zeta)P \qquad t > 0 \qquad 0 < z < d \tag{1a}$$

$$T(0,z) = T_0 \qquad z = 0, \ -k\frac{\partial T}{\partial z} = h\,(T_e - T) + \zeta q_0 \qquad z = d, \ \frac{\partial T}{\partial z} = 0 \tag{1b}$$

where ρc and k are the volumetric heat capacity and thermal conductivity of ice, $T_e(t)$ is the temperature of air outside the boundary layer, $h(t)$ is the convective heat transfer coefficient. The adiabatic condition at $z = d$ means that we neglect heat transfer by conduction at $z > d$. The value of ζq is the radiative flux absorbed in a thin surface layer, whereas the term $(1-\zeta)P(z)$ is the absorbed power of the short-wave radiation, where the coefficient $\zeta < 1$ is determined as:

$$\zeta(\lambda_*) = \int_{\lambda_*}^{\lambda_{22}} I_{\lambda,b}(T_{sol})\,d\lambda \Big/ \int_{\lambda_{11}}^{\lambda_{22}} I_{\lambda,b}(T_{sol})\,d\lambda \tag{2}$$

where $I_{\lambda,b}(T_{sol})$ is the Planck function at the Sun "radiative" temperature $T_{sol} \approx 6000$K. The total absorbed radiation power, $P(z)$, and the physical sense of the wavelengths λ_{11}, λ_{22}, and λ_* are discussed below.

The spectral properties of pure ice without any inclusions were discussed in the article *Spectral optical properties of pure ice and snow*. The geometrical optics can be used to study the optical properties of gas bubbles with large diffraction parameter. Small gas bubbles in ice are usually spherical. The bubbles elongated by strain return to their spherical shape by diffusive processes (Alley and Fitzpatric 1999). Therefore, only spherical bubbles are considered below. For simplicity, it is assumed that the bubbles have the same radius a, which is much greater than the wavelength. The absorption coefficient and transport scattering coefficient of ice containing such bubbles can be calculated as follows (Dombrovsky 2004):

$$\alpha_\lambda = (1 - f_\mathrm{v})\alpha_\lambda^0 \qquad \sigma_\lambda^\mathrm{tr} = \sigma_\lambda(1 - \bar{\mu}_\lambda) = 0.75 f_\mathrm{v}\, Q_\mathrm{s}^\mathrm{tr}/a \tag{3}$$

where f_v is the volume fraction of bubbles, $\alpha_\lambda^0 = 4\pi\kappa/\lambda$ is the absorption coefficient of ice without bubbles. It is interesting to note that the above equations agree totally with a simple physical approach applicable to many optically soft semi-transparent materials of different nature. This approach is based on the following principles formulated by Dombrovsky and Baillis (2011):

1. The absorption coefficient is independent of the material morphology. It is directly proportional to the volume fraction of the absorbing substance in a porous material: $\alpha_\lambda = \alpha_\lambda^0(1 - p)$, where α_λ^0 is the absorption coefficient of bulk material of the same chemical composition and p is the porosity.
2. The scattering is insensitive to a weak absorption and can be predicted by an analysis of the material morphology.

There are many materials which optical properties satisfy the above principles. Some examples can be found in (Dombrovsky et al. 2005, 2007, Ganesan et al. 2013, Baillis et al. 2013, Hakoume et al. 2014, Lisitsyn et al. 2016, Tuchin 2017). A relative mutual independence of absorption and scattering is not a specific property of semi-transparent porous materials. According to the Rayleigh–Gans theory (Bohren and Huffman 1983), this is typical of media containing optically soft particles.

Obviously, $f_v << 1$ in the case of ice containing small bubbles and one can consider α_λ^0 instead of α_λ in this particular problem. The absorption coefficient of ice is very small in the near-ultraviolet and visible ranges but increases strongly with the wavelength. So, the ice layer of thickness more than 1–2 mm is almost opaque in the wavelength range of $\lambda > 1.4$ µm.

Let us consider the effect of bubbles on the transport efficiency factor of scattering of ice at the wavelength $\lambda < 0.8$µm, where the scattering is more important. The index of refraction of ice in this spectral range is not close to unity. Therefore, the approximation of optically soft host medium (when $n - 1 << 1$) is not appropriate. The Mie theory calculations reported by Dombrovsky (2004) and Dombrovsky et al. (2005) showed that absorption has almost no effect on scattering in a weakly absorbing host medium, and it is sufficient to consider the case of nonabsorbing medium to determine the transport efficiency factor of scattering. The calculations at indices of refraction from $n = 1.2$ to 1.5 showed that Q_s^{tr} is almost constant at the diffraction parameter $x > 20$:

$$Q_s^{tr} = 0.9\,(n-1) \tag{4}$$

The resulting equations for the absolute and relative scattering coefficients are:

$$\sigma_\lambda^{tr} = 0.675\,(n-1)\,f_v/a \qquad \sigma_\lambda^{tr}/\alpha_\lambda = 0.3375 f_v\,(n-1)/(\kappa x) \tag{5}$$

Obviously, the scattering is predominant at $f_v > 10\kappa x$. Note that the asymmetry factor of scattering is the same for all large bubbles and can be approximated as:

$$\bar{\mu}_\lambda = 1 - 0.45(n-1) \tag{6}$$

The above results for scattering and the effect of bubbles on radiation absorption have been used by Dombrovsky et al. (2005) for fused quartz containing gas bubbles, in radiative heat transfer analysis for water containing steam bubbles (Dombrovsky 2004, 2007, 2009, Dombrovsky et al. 2009, Dombrovsky and Davydov 2010), and as a limiting solution for hollow glass microspheres in a polymer matrix (Dombrovsky 2005, Dombrovsky et al. 2007).

As a verification of the above analytical approximation, one can consider the results of more accurate calculations ("without the phase interferences") at $n = 4/3$ reported by Wu et al. (2007):

$$Q_s = 1.9997 \qquad \bar{\mu}_\lambda = 0.8444 \tag{7}$$

Note that the above value of $\bar{\mu}_\lambda$ is exactly the same as that obtained analytically by Warren et al. (2006) and one can obtain $Q_s^{tr} = 0.311$, whereas Eq. (5) gives $Q_s^{tr} = 0.3$. At the same index of refraction, Eq. (6) gives $\bar{\mu}_\lambda = 0.85$ which is also close to the exact value. This solution is applicable only in the spectral ranges where the host medium is semi-transparent. In the opacity range, where the medium is strongly absorbing, the radiative transfer problem degenerates and there is no any reason to study the volumetric optical properties. It means that heat transfer modeling should be based on quite different approaches for the spectral regions of the medium semi-transparency and opacity.

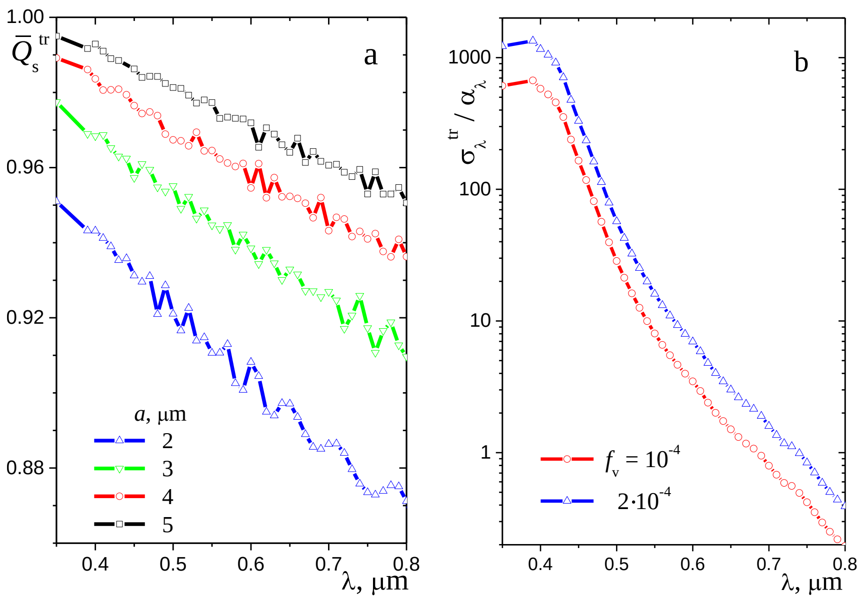

Figure 1. Mie theory calculations of for ice containing bubbles with radius 50μm.

Consider now some results of spectral calculations using the Mie theory for small bubbles. The value of $\bar{Q}_{\mathrm{s}}^{\mathrm{tr}}$, which is a ratio of $Q_{\mathrm{s}}^{\mathrm{tr}}$ to the approximate value given by Eq. (4), is presented in Fig. 1a. Most likely, Eq. (4) is applicable in the wavelength range of $\lambda < 0.8$μm for all the bubbles with radius of $a > 10$ μm. Moreover, the error of this equation is less than 5 % even at $a = 5$ μm. In the case of more realistic radius of bubbles, the transport scattering coefficient of ice with bubbles is greater than the absorption coefficient (see Fig. 1b).

Note that the above optical model is based on the hypothesis of independent scattering of light by single bubbles. The error of this approach is negligible when both the size of randomly positioned bubbles and the distances between them are much greater that the wavelength of solar light.

According to Eq. (5), the only parameter that affects the light scattering in ice is the "scattering parameter" $S = f_{\mathrm{v}}/a$ measured in m^{-1}. This simplifies the analysis of computational results. The calculated profiles of $\bar{p}(z,\lambda) = \alpha_\lambda(z)\bar{G}_\lambda(z)$ ($\bar{G}_\lambda$ is the spectral irradiance) in the visible range are shown in Fig. 2. The strong increase in the index of absorption of ice with the wavelength is responsible for the absorption of radiation even in the range of $0.6 < \lambda < 0.7$ μm. The increase in both the wavelength and scattering parameter leads to a smaller penetration depth of incident radiation.

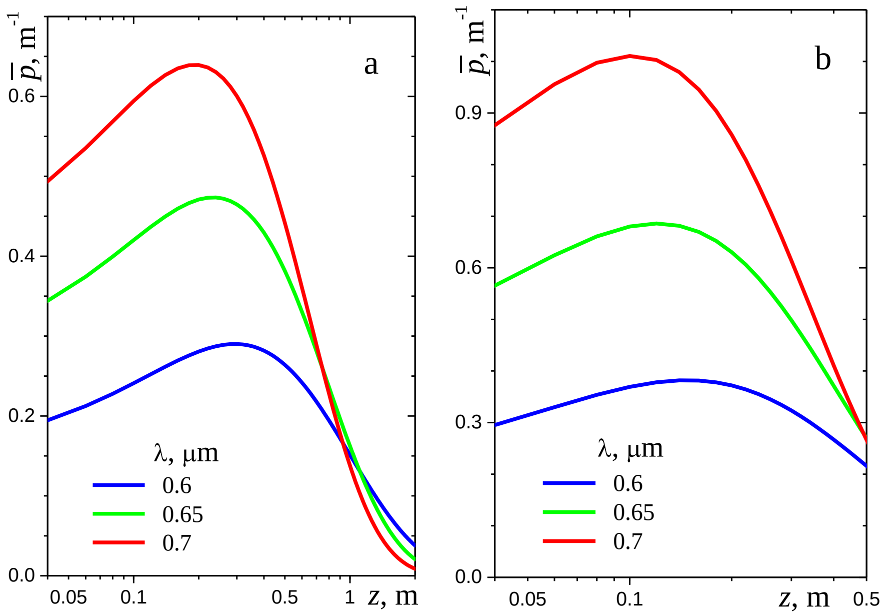

Figure 2. Normalized profiles of absorbed spectral radiation power: a – $S = 20$ m^{-1}, b – 50 m^{-1}.

Relative contributions of different spectral ranges to the solar radiation absorption in ice with bubbles can be estimated assuming that the spectrum of solar radiation is similar to that of the blackbody at temperature T_{sol}. It is sufficient to compare the following two profiles:

$$\bar{P}_1(z) = \frac{1}{P_0}\int_{\lambda_{11}}^{\lambda_{12}} \bar{p}(z,\lambda)\, I_{\lambda,\mathrm{b}}(T_{\mathrm{sol}})\,\mathrm{d}\lambda \qquad \bar{P}_2(z) = \frac{1}{P_0}\int_{\lambda_{12}}^{\lambda_{22}} \bar{p}(z,\lambda)\, I_{\lambda,\mathrm{b}}(T_{\mathrm{sol}})\,\mathrm{d}\lambda \tag{8}$$

where $P_0 = \int_{\lambda_{11}}^{\lambda_{22}} I_{\lambda,\mathrm{b}}(T_{\mathrm{sol}})\,\mathrm{d}\lambda$, $\lambda_{11} = 0.35\mu\mathrm{m}$, $\lambda_{12} = 0.8\mu\mathrm{m}$, $\lambda_{22} = 2.778\mu\mathrm{m}$. The function $\bar{P}_1(z)$ corresponds to a contribution of the near-ultraviolet and visible ranges, whereas $\bar{P}_2(z)$ characterizes a contribution of the near-infrared range. Fig. 3a indicates that infrared radiation contributes strongly to the absorption of solar radiation in a thin surface layer of the ice sheet, whereas the absorption of radiation at distances more than about 25 cm from the surface is determined by a contribution of the short-wavelength range. The latter effect depends on the scattering parameter. Note that the absorbed radiation power presented in Fig. 3b was calculated at the approximate value of $q_0 = 1$ kW/m^2 for the incident radiative flux at the ice sheet surface.

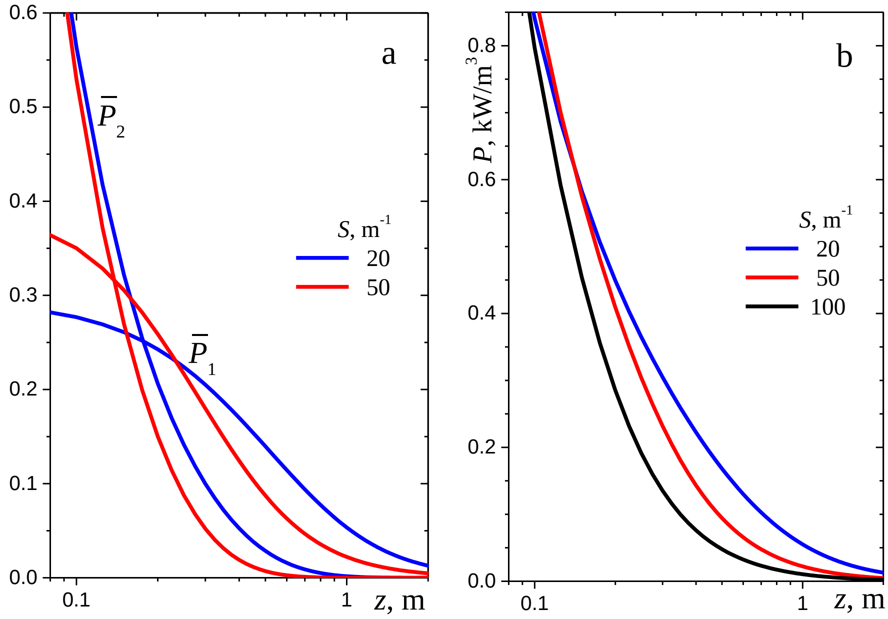

Figure 3. Absorption of solar radiation in ice sheet containing gas bubbles:

a – relative contribution of two spectral ranges, b – profiles of total absorbed power.

The calculations show that the radiation at $\lambda = 1.4$ μm is almost totally absorbed in the surface layer of thickness $d_{\mathrm{surf}} = 1$ mm, whereas $d_{\mathrm{surf}} = 1/\alpha_\lambda \approx 0.1$ mm at $\lambda = 2.5$ μm. It means that the heat transfer analysis can be based on the assumption that the infrared radiation at $\lambda > 1.4$ μm is absorbed just on the surface of ice sheet. This makes possible a separate consideration of the radiation absorbed at the surface and in the volume of ice sheet.

It was found in (Dombrovsky and Kokhanovsky 2020) that specific spectral properties of ice containing gas bubbles affect considerably the normal-hemispherical reflectance, $R_{\text{n-h}}$, of an ice sheet. First of all, it should be noted that the spectral dependences of $R_{\text{n-h}}$ in the visible range are strong and quite different from those for albedo of pure and polluted snow. It is also interesting that the scattering parameter of a thick ice sheet not covered by snow can be estimated using the measurements of $R_{\text{n-h}}$ in the wavelength range of $0.5 < \lambda < 0.65$ μm.

In the heat transfer calculations, the values of $\lambda_* = 1.4$ μm, $\zeta = 0.118$, and the constant thermal properties $k = 2.3$ W/(m K) and $\rho c = 1.84$ MJ/(m^{3}K) are used. It was assumed that $T_0 = -10$ $^{\text{o}}$C, $T_{\text{e}} = -20$ $^{\text{o}}$C, and $h = 20$ W/(m^2 K) (this corresponds to a wind of speed about 4 m/s). The numerical calculations were performed for the solar irradiation of ice sheet of thickness $d = 1$ m during three hours. The results obtained at various values of the scattering parameter are presented in Fig. 4.

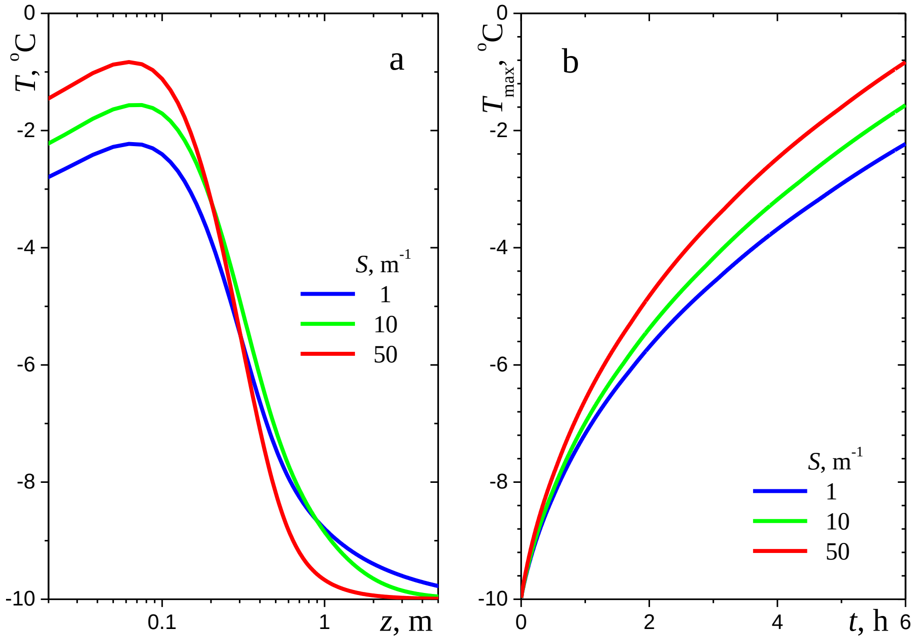

Figure 4. Effect of scattering parameter on (a) temperature profiles in ice sheet at $t = 6$h and (b) the maximum temperature of ice.

The convective cooling of ice sheet by a wind of relatively cold air leads to the fact that the maximum temperature is not on the surface of irradiated ice sheet, but at a distance of about 6–7 cm below the surface. The position of this maximum of temperature is weakly sensitive to the scattering parameter, but the value of T_{max} increases considerably with S and may reach the melting temperature in about 7 h at $S = 50$ m^{-1}. Fig. 4 indicates that gas bubbles in ice affect considerably the internal overheating of ice sheet. At the same time, it should be mentioned that the model problem is very simplified. The most important assumption is the absence of a snow layer at the ice sheet surface. The highly scattering snow cover will prevent penetration of visible radiation to the ice sheet. It this case, the effect of gas bubbles on solar heating of ice will be not as strong as that in the above model problem.

It is known that a collimated irradiation of semi-transparent scattering media leads to considerable volumetric absorption of the radiation. Moreover, the maximum of the radiation absorption can be observed at a distance from the irradiated surface. This effect is explained by a relatively strong absorption of the diffuse radiation formed due to multiple scattering in the medium. In the case of a scattering ice sheet, the internal maximum of temperature leads to tensile stresses at the sheet surface. There are several cases when the relatively warm internal ice layers may lead to surface cracks on the ice sheet surface. The known example is the cracking of sea ice when the air temperature drops below the water temperature under the floating ice sheet. The optical properties of light-scattering ice affect the periodic heating and cooling of a surface layer of glaciers that is important for their complex behaviour.

The theoretical estimates showed that small gas bubbles may lead to a considerable additional heating of ice at several centimeters below the ice-sheet surface. The reported computational model is considered as a basis of more comprehensive modeling of solar heating of various ice sheets with account for the effect of a snow cover and specific properties of continental ice and sea ice. In particular, in some cases one also needs to account for other absorbing and scattering particulate matter in ice such as brine, microalgae, soot particles, and dust inclusions.

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